Multiple Discriminant Analysis of Corporate Bankruptcy

In this paper, corporate bankruptcy is analyzed by employing the predictive tool of multiple discriminant analysis. Using several firm-specific metrics gauging qualitative and quantitative characteristics of a company, the multiple discriminant analysis successfully predicts bankruptcy up to five years in advance for a sample study of 111 firms. Further applications of multiple discriminant analysis in quantitative value investing and portfolio design strategy are also discussed, and potential areas of continued research are proposed.

Background

For any corporation or firm, a comprehensive and clear understanding of the business’s current and projected performance is crucial on both an internal and external level. Clients and employees alike are interested in how the firm’s profits, returns, and other significant indicators not only demonstrate its prevailing wellbeing, but also its potential for either failure or continued success. In this paper, we examine the predictability of bankruptcy in firms. The implementation of particular tools to measure a company’s health has become a crucial element of quantitative investment research. However, the question of how exactly a company's wellbeing should be modeled, as well as the quantitative and qualitative effectiveness of any such tools present various complications in the analysis. After considering regressions across market history, we have focused on solvency metrics, debt metrics, efficiency metrics, and profitability metrics, as they continue to have a high degree of predictive power in indicating company failure or success.
Multiple Discriminant Analysis

Reliance upon ratio analysis has historically drawn criticism for its overall univariate approach and consequential inability to fully address the correlations and interactions between potential indicators. Multiple discriminant analysis is another (perhaps more effective tool) that makes predictions in a particular context based on generally qualitative observations. Multiple discriminant analysis has been used across a spectrum of disciplines, from behavioral sciences to consumer credit evaluations. Numerous advantages arise when using multiple discriminant analysis, as opposed to ratio analysis. Multiple discriminant analysis enables a comprehensive analysis of multiple characteristics of a firm and their interactions, while ratio analysis solely permits a valuation of the indicators one at a time. Furthermore, multiple discriminant analysis reduces the number of variables from N to N-1 groups, where N is the number of initial groups. With this paper’s particular focus on bankruptcy, the analysis occurs in only one dimension, as the two initial groups are the bankrupt and non-bankrupt firms. In this paper, we will implement multiple discriminant analysis to prioritize indicators of firm bankruptcy, determine the effect of each ratio in the analysis, and establish a method for measuring such effects.
Formula

If we take all of the potential indicators, call them variables y1, y2, y3,...yn and assign the calculated discriminant coefficients a1, a2, a3,...an, we will establish a discriminant function formula that combines all the indicators and coefficients to equal an ultimate discriminant value, X.

\[ X = a_1y_1 + a_2y_2 + a_3y_3 + \ldots + a_ny_n \]

Analysis of firms and their potential for bankruptcy will be analyzed using this discriminant function, and thus X will determine whether or not the firm will ultimately reach bankruptcy.

Variables

The variables included in the analysis are five financial metrics that would otherwise be analyzed in the univariate method of financial ratio analysis. These five metrics are not necessarily the most statistically significant independently, but are selected through measuring the statistical significance of alternative functions, evaluating the relative contributions of each variable, and evaluating the correlations among the variables. Using an F-test for equality of variance for each metric, the variables were generally deemed discriminating between the bankrupt and non-bankrupt groups. The determinant function thus includes the following properties of any firm: a liquidity metric, a cumulative profitability metric, a productivity metric, a solvency metric, and an efficiency metric.
The liquidity metric is a critical property of a firm to include and analyze, as it measures the firm’s capacity to cover its financial liabilities in the short term. Generally, those companies experiencing losses also experience diminishing present assets in comparison to total assets. As a result, the metric in the discriminant function would be expected to decrease for those firms more likely to face bankruptcy.

The cumulative profitability metric measures how much profit a firm makes on a relative scale to its total assets. This ratio is also unique in that it is affected by the particular age of a company – those companies more recently established are more likely to have lower retained earnings than older businesses.

The productivity metric measures earnings prior to any tax or interest reductions, thereby demonstrating how well a firm can generate earnings from its assets before any fees or obligations must be taken into account.

The solvency metric marks a boundary between a firm’s insolvency and solvency. By relating the market value of equity and total debt, the metric determines how much the firm’s total equity and total debt can decline in value before the liabilities surpass the total assets (equity and total debt).

The efficiency metric demonstrates a firm’s assets’ ability to generate sales. This property is uniquely important for multiple discriminant analysis, as the metric becomes even more significant when analyzing its interactions with the other properties at once in a firm-competitive context.
As a general summary of the five properties, a firm that is less likely to experience bankruptcy should experience higher ratios for each of the properties.

We expect these coefficients to be positive, and thus, the higher the X, the healthier the company.

Results

In a study of 111 sample firms divided by each group as discussed above (one group includes 53 public companies that filed for bankruptcy between 1969-1975, and the other group includes a 58 paired sample of public companies that still existed in 1976), a multiple discriminant analysis employed the five aforementioned properties and calculated each companies’ respective discriminant function and value (X).

Much like a regression model with R2 measuring the predictive power of the indicators, a percentage was calculated of the total correct classifications of a firm (either bankrupt or not bankrupt) divided by the total number of firms (66).

The first classification percentage was calculated for the 111 firms themselves, using the data that we used to derive the original discriminant function. Thus, we expect a very high percentage. Results show that 92.8% classifications were indeed correct.

The next percentage was calculated using data from the firms two years prior to bankruptcy. 89% of the classifications were correct, indicating the predictive capability of the discriminant function even two years in advance.
To see how far back the multiple discriminant analysis could be utilized in predicting bankruptcy, percentages were also calculated for firms three, four, and five years prior. The results were 83.5%, 79.8%, and 76.8%, respectively. Past 5 years prior to bankruptcy, the percentages indicate the increasing temporal limitations of our model to a maximum of five years prior to filing for bankruptcy.

In another study, 86 distressed firms from 1969-1975, 110 bankrupted companies from 1976-1995, and 120 bankrupted companies from 1997-1999 were analyzed. The calculated accuracy percentage was between 82% and 94%.

Aside from the percentages, the discriminant value (X) itself was calculated for each firm as well. Results showed that firms with X values above 2.99 will fall into the non-bankrupt group, while firms with X values below 1.81 are all bankrupt. To reconcile the range of X values between 1.81 and 2.99, the midpoint of a calculated critical value interval is used as the best discriminating marker between bankruptcy and non-bankruptcy: 2.675.

In the model, Type I errors occur when the X value does not predict bankruptcy, even though bankruptcy ultimately occurs. Type II errors occur when the X value predicts bankruptcy, and the firm does not experience bankruptcy.

Further Applications

Use of the multiple discriminant model of analysis has also been shown to be highly predictive in appraising particular investment criteria for both
portfolio and stock strategy. For quantitative value investing, an investor has the ability to potentially prevent making poor investment decisions by anticipating any declines in the relevant firms. If an investor is cognizant of such future declines, he should sell his stock in order to avoid further falling prices. Such a financial decision would be preventative of increased losses and provide greater capital for the individual to make alternative investment decisions.

Further areas of research to explore in the analysis of corporate bankruptcy could be an examination of non-US firms, smaller, unincorporated firms, and firms across a wider range of industries. Another area for further research could be exploring multiple levels of bankruptcy to account for the associated, more realistic risks involved in bankruptcy, as opposed to the dichotomous classification of bankrupt vs. non-bankrupt firms.